Math 221: Test 2 Review Sheet

This document tells you everything that you will need to know in order to do well on your first test. Each bullet is a requirement that you should ensure that you satisfy prior to taking the test. Not all bullets have problems following them, but these concepts are just as important.

- Know the definitions of all related terms to the various concepts covered in order to solve problems that use them. There will be no definitions question on this test.

Arithmetic Models

- Given a word problem, determine which model is being illustrated.
  - Addition: Set Model, Number Line
  - Subtraction: Take-Away, Missing Addend, Comparison, Number Line
  - Multiplication: Repeated Addition/Number Line, Array/Area, Cartesian Product
  - Division: Partition, Measurement

  Arithmetic Models: Determine which model is being described in each problem, and explain your answer. These are all of the problems you were given as examples, and the test questions will be similar.

  1. Susan ate 56 donuts in two months. If you ate 6 donuts in the second month, how many donuts did you eat in the first month?
  2. You have 10 different pairs of socks that go well with any shoe and you have 1,394 pairs of shoes. How many ways can you fashionably cover your feet?
  3. I have 16 colored T-shirts and 8 button up shirts. How many ways can I combine them for one outfit?
  4. A base ten flat is actually composed of units. Each row has 10 units and each column has 10 units. How many units are in a flat?
  5. A cashier at a grocery store tells her customer that his total is $25.89. The customer gives the cashier $4.00 in cash and has to pay the rest on a debit card. How much money does the customer pay on his debit card?
  6. You have 9 apples and you give 4 away to your teachers. How many apples do you have left?
  7. Little Jimmy got 6 toy cars and 3 toy trucks for Christmas. How many toy vehicles did he get altogether?
  8. James bought a new camera. He took pictures of birds and squirrels at the park. He took 63 pictures of birds and 24 pictures of squirrels. How many pictures did he take in all?
  9. If you were on the fourth floor of a building and moved up 3 floors, what floor are you now on?
  10. You have a quilt that is made of 1 foot by 1 foot squares. If the quilt has 6 rows with 10 squares in each row, how many squares go into the quilt?
  11. Each year for 3 years, you swim the 21 miles across the English channel. How many miles did you swim altogether?
  12. You have a box of 50 envelopes and you use 3 to mail off letters. How many envelopes do you have left?
  13. You bring 56 cookies to class and you want to give cookies to all of your 28 students. If you give each student an equal number of cookies, how many cookies does each student get?
  14. Your cat weighed 15 pounds and lost 2 pounds last month due to a much needed diet. How much does he weigh now?
  15. Karen and Josh were picking strawberries. Karen picked 226 strawberries. Josh picked 193 strawberries. How many strawberries did they pick altogether?

16. You go strawberry picking and you have 6 strawberries. If you pick 5 more strawberries, how many do you have?
17. A refrigerator magnet weighs 3 ounces. If you have 4 of these, how much do they weigh altogether?
18. You drove 100 miles in a two hour period. If you drove 40 miles in the second hour, how many miles did you drive during the first?
19. You have 3 dollars and you want to buy a book that costs 7 dollars. How much more money do you need to buy the book?
20. Latoya goes on a two hour shopping spree and spends $1500 in two hours. If she spent $685 in the second hour, how much did she spend during the first hour?
21. Jennas cookie jar had 16 cookies in it. She baked 24 more cookies. When they cooled, she added them to the cookie jar. What is the new total number of cookies in Jennas cookie jar?
22. Michael is driving the 300 mile trip to Virginia. If he has driven 90 miles, how many more miles does he have to drive?
23. A king size candy bar costs $1. If you have a huge craving for chocolate and buy five of these, how much do you spend?
24. If a sheet of paper is 10 in. by 12 in., what is the area of the sheet of paper?
25. While planning a field trip for the whole 3rd grade, you decide that you'll need a chaperone for every 5 students. If there are 315 3rd graders at your school, how many chaperones do you need?
26. Your room is 12 ft by 14 ft. What is the area of your floor?
27. You have 4 different styles of coffee cups and 3 different blends of coffee. In how many ways can you have a cup of coffee?
28. Your cat weighed 8 pounds and gained 2 pounds last month. How much does he weigh now?
29. Nick and his friends ate 285 M&Ms from a bowl containing 546 M&Ms. How many more M&Ms do they need to eat in order to eat them all?
30. Water is flowing out of the faucet at 300 mL per second for 30 seconds. How much water comes out of the faucet?
31. You are saving up to buy a $8000 used car that you found in the paper. If you have $5500 now, how much more money do you need to earn in order to buy the car?
32. Bob has 1,297 football cards. Sammy has 1,488. How many more cards does Sammy have than Bob?
33. You have 1 laptop and 2 desktop computers in your house. How many computers do you have total?
34. You ate 2 pieces of cake and your friend ate 5 pieces of cake. How many more pieces of cake did your friend eat than you did?
35. You have 3 cats and 4 dogs. How many animals do you have total?
36. If you have a sheet of stickers with 15 rows of 10 stickers, how many stickers are on the sheet?
37. If you drive down the interstate at 70 mph for 3 hours, how far did you go?
38. Justin read 51 pages of his book on Monday. On Tuesday, he read 38 more pages. How many pages did he read on both days combined?
39. You earned 100 points on the last test and the student next to you earned 90. How many more points did you earn than the student next to you?
40. You buy 500 pieces of Halloween candy and plan to give 4 pieces to each trick-or-treater. How many trick-or-treaters can you serve?

- Determine whether a given set has the closure, commutative, associative, identity properties over the 4 basic operations.

Set Properties: Determine whether the following sets have the closure, commutative, associative, or identity property over the given operation.

1. $\{0, 2, 4, 6, 8, ...\}$ over $+$
2. $\{1, 2, 3, 4, 5...\}$ over $+$
3. $\{0, 1\}$ over $-$
4. $\{0, 1\}$ over $\times$
5. $\{1, 4, 9, 16, 25, 36, 49, ...\}$ over $\times$
6. $\{1, 2\}$ over $\div$
7. $\{0, 1, 2, 3, 5, 8, 13, 21, 34, ...\}$ over $+$ (each new number comes from adding the previous two)
8. $\{2, 3, 6, 18, 108, ...\}$ over $\times$ (each new number comes from multiplying the previous two)

Base 5 Arithmetic

- Perform Base 5 addition using Base 5 blocks, the standard algorithm, and the lattice algorithm.

  Base 5 Addition: Perform the following addition problems using Base 5 blocks, the standard algorithm, and the lattice algorithm.

  1. $213_{\text{five}} + 133_{\text{five}}$
  2. $241_{\text{five}} + 404_{\text{five}}$
  3. $3444_{\text{five}} + 444_{\text{five}}$
  4. $1132_{\text{five}} + 1334_{\text{five}}$
  5. $2430_{\text{five}} + 1322_{\text{five}}$
  6. $2011_{\text{five}} + 2341_{\text{five}}$

- Perform Base 5 subtraction using Base 5 blocks, the standard algorithm, and the equal additions algorithm.

  Base 5 Subtraction: Perform the following subtraction problems using Base 5 blocks, the standard algorithm, and the equal additions algorithm.

  1. $213_{\text{five}} - 133_{\text{five}}$
  2. $401_{\text{five}} - 244_{\text{five}}$
  3. $2430_{\text{five}} - 1322_{\text{five}}$
  4. $4330_{\text{five}} - 1432_{\text{five}}$
  5. $3000_{\text{five}} - 1444_{\text{five}}$
  6. $4103_{\text{five}} - 3114_{\text{five}}$

- Perform Base 5 multiplication using Base 5 blocks, the standard algorithm, and the lattice algorithm.

  Base 5 Multiplication: Perform the following multiplication problems using Base 5 blocks (if both numbers 2 or less digits), the standard algorithm, and the lattice algorithm.

  1. $13_{\text{five}} \times 24_{\text{five}}$
  2. $43_{\text{five}} \times 41_{\text{five}}$
  3. $124_{\text{five}} \times 44_{\text{five}}$
  4. $323_{\text{five}} \times 30_{\text{five}}$
  5. $431_{\text{five}} \times 341_{\text{five}}$
  6. $3214_{\text{five}} \times 1443_{\text{five}}$

- Perform Base 5 division using Base 5 blocks and the long division algorithm.

  Base 5 Division: Perform the following division problems using Base 5 blocks (if the divisor is a single digit) and the long division algorithm.

  1. $33_{\text{five}} \div 2_{\text{five}}$
  2. $131_{\text{five}} \div 3_{\text{five}}$
  3. $2242_{\text{five}} \div 4_{\text{five}}$
  4. $434_{\text{five}} \div 11_{\text{five}}$
  5. $3021_{\text{five}} \div 31_{\text{five}}$
  6. $4411_{\text{five}} \div 123_{\text{five}}$

- Explain why these various algorithms work.

  The Standard Algorithms: Use base 5 blocks to explain the regrouping steps in the following base 10 addition, subtraction, and multiplication problems.

  1. $846 + 321$
  2. $599 + 101$
  3. $207 - 82$
  4. $4024 - 1251$
  5. 1885×4 (use repeated addition)
  6. 24×18 (use array)

  The Lattice Algorithms: Explain how the diagonals are automatically regrouping place values in the following addition and multiplication problems.

  1. $846 + 321$
  2. 24×18
  3. 152×67

  The Equal Additions Algorithm: Explain how we choose what to add in the following subtraction problems, and explain why this doesn't change the problem.

  1. $207 - 82$
  2. $4024 - 1251$

  Long Division Algorithm: Use the partition method to show how the long division algorithm works for the following base 10 division problems. Explain your steps carefully.

  1. $46 \div 2$
  2. $621 \div 3$
  3. $1268 \div 5$

4.1

- Perform divisibility tests for 2, 3, 4, 5, 6, 7, 8, 9, 10, and 11 to find which numbers divide a given number.

  Divisibility Tests: Determine whether 2, 3, 4, 5, 6, 7, 8, 9, 10, and 11 divide the following numbers.

  1. 540
  2. 3465
  3. 1848
  4. 2873
  5. 27720
  6. 103818

4.2

- Determine is a number is prime or composite.

  Prime or Composite Numbers: Determine if the following numbers are prime or composite.

  1. 169
  2. 203
  3. 211
  4. 221
  5. 1547
  6. 2311

- Perform the Sieve of Eratosthenes to find all of the primes up to a given number.

  Sieve of Eratosthenes: Find all of the prime numbers up to 400. (Note: $\sqrt{400} = 20$, so you only have to cross out multiples of the primes up to 19.)

- Find the prime factorization of a number (which exists uniquely by the Fundamental Theorem of Arithmetic) using a factor tree.

Fundamental Theorem of Arithmetic: Find the prime factorization of the following numbers.

1. 540
2. 3465
3. 1848
4. 2873
5. 27720
6. 103818

4.3

- Use the Intersection of Sets and Prime Factorization Methods to find the Greatest Common Divisor (GCD) and Least Common Multiple (LCM) of two numbers.

Greatest Common Divisor and Least Common Multiple: Find the GCD and LCM of the following numbers using Intersection of Sets (if practical) and Prime Factorization.

1. 4 and 6
2. 12 and 18
3. 20 and 32
4. 98 and 147
5. 90 and 84
6. 1800 and 60

Helpful Rules/Formulas:

1. The set model of addition is used to represent the addition of two numbers by two disjoint sets.
2. The number line model of addition is used to represent a second number of the same time being added to the first.
3. The take-away model of subtraction is used to represent starting with a set and removing elements.
4. The missing addend model of subtraction is used to represent when you are determining how much is left to get to a certain number.
5. The comparison model of subtraction is used to represent when you have two sets and you want to find how many more elements are in one set than another.
6. The number line model of subtraction is used to represent when you are given a sum and the number that was added to get there, and you want to find out what the original number was.
7. The repeated addition model of multiplication is used to represent when you are adding up a number multiple times.
8. The number line model of multiplication is used to represent when you are adding on to a number by the same amount multiple times.
9. The array model of multiplication is used to represent when you are counting up items arranged in rows and columns.
10. The area model of multiplication is used to represent when you are finding the area of something given lengths on two perpendicular sides.
11. The cartesian product model of multiplication is used to represent when you are finding the number of combinations that can be made between items in two sets.
12. The partition model of division is used to represent when you are equally distributing a number of elements into smaller groups and finding how many can be put in each group.
13. The measurement model of division is used to represent when you are equally distributing a number of elements with a set amount in each group and finding out how many groups can be made.
14. (Closure Property) If a and b are in the set, then $a * b$ (where $*$ is any of the 4 basic operations) is also in the set.
15. (Commutative Property) If a and b are in the set, then $a * b = b * a$.
16. (Associative Property) If a, b, and c are in the set, then $(a * b) * c = a * (b + c)$.
17. (Identity Property) There is a unique number e in the set (0 for addition, 1 for multiplication), the identity, such that for any a in the set, $a * e = e * a = a$.

18. (Zero Product Property) For any real number a, $a \times 0 = 0 \times a = 0$.
19. (Distributive Property) For any real numbers a, b, and c, $a(b+c) = ab + ac$ and $a(b-c) = ab - ac$.
20. Anytime we "regroup" in addition or multiplication, we are actually making an exchange to the next place value.
21. The lattice algorithms automatically group the proper place values. In the case of multiplication, this is partially due to noting which place the various parts of each number lie. For example, when we multiply 64×3, the 6×3 row is actually 60×3.
22. The equal additions algorithm works because we are actually adding and subtracting the same value.
23. The long division algorithm carefully partitions elements into the number of groups given by the divisor. When we divide, we decide how many of that place value we can place into our groups. When we multiply, we determine how many we used up. When we subtract, we determine how much is left. And finally, when we bring down, we are making an exchange to a smaller unit.
24. $2 \mid n$ if and only if $2 \mid$ the last digit of n.
25. $3 \mid n$ if and only if $3 \mid$ the sum of the digits of n.
26. $4 \mid n$ if and only if $4 \mid$ the last two digits of n.
27. $5 \mid n$ if and only if n ends in 0 or 5.
28. $6 \mid n$ if and only if $2 \mid n$ and $3 \mid n$. In general, if a number can be written as the product of two relatively prime numbers, we can do a similar divisibility test with these two numbers.
29. $7 \mid n$ if and only if $7 \mid$ the number formed by removing the last digit of n, then subtracting twice that digit from the result.
30. $8 \mid n$ if and only if $8 \mid$ the last three digits of n. In general, powers of 2 will use the last number of digits of n as the exponent of 2 that makes that number.
31. $9 \mid n$ if and only if $9 \mid$ the sum of the digits of n.
32. $10 \mid n$ if and only if n ends in 0.
33. $11 \mid n$ if and only if $11 \mid$ the number formed by alternatively adding and subtracting the digits of n.
34. To check if a number n is prime, we determine whether the primes 2, 3, 5, etc. divide it. If no prime smaller than $\sqrt{n}$ divides it, then it is prime.
35. The Sieve of Eratosthenes is performed by writing all the natural numbers up to a given number, then crossing out all multiples of the primes up to the square root of the top number. The numbers that are not crossed out are all prime.
36. The intersection of sets method to find the GCD involves writing the set of all divisors of each number, circling the common ones, and choosing the largest of the common ones as the GCD.
37. The intersection of sets method to find the LCM involves writing sets of all multiples of each number until they have a common one. The smallest common multiple is the LCM.
38. The prime factorization methods for GCD and LCM involve finding the prime factorizations of each number. To find the GCD, take the smaller power of any prime that shows up in both, and multiply these together. To find the LCM, take the highest power of any prime that shows up in either number, and multiply these together.